

Differentiation of Trigonometric Functions

If we look at the slope of the tangent lines at the five key points for the sine curve, it can be shown that the derivative is the cosine function. You must memorize these two derivatives.

$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\cos x] = -\sin x$
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Let's derive the other four with the help of the trig identity: $\sin^2 x + \cos^2 x = 1$.

EX #1: $y = \tan x$

Rewrite:
 $y = \frac{\sin x}{\cos x}$

$$y' = \frac{\cos x \cos x - \sin x (-\sin x)}{\cos^2 x}$$

$$y' = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$y' = \frac{1}{\cos^2 x} \Rightarrow \underline{\underline{\sec^2 x}}$$

EX #3: $y = \csc x$

$$y = \frac{1}{\sin x} \Rightarrow y' = \frac{-\cos x}{\sin^2 x}$$

$$y' = \frac{-\cos x}{\sin x \cdot \sin x}$$

$$y' = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$$

$$\underline{\underline{y' = -\csc x \cot x}}$$

EX #2: $y = \cot x$

Rewrite $y = \frac{\cos x}{\sin x}$

$$y' = \frac{\sin x (-\sin x) - \cos x (\cos x)}{\sin^2 x}$$

$$y' = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$y' = \frac{-1}{\sin^2 x} \Rightarrow \underline{\underline{-\csc^2 x}}$$

EX #4: $y = \sec x$

$$y = \frac{1}{\cos x} \Rightarrow y' = \frac{-(-\sin x)}{\cos^2 x}$$

$$y' = \frac{\sin x}{\cos x \cdot \cos x}$$

$$y' = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$$

$$\underline{\underline{y' = \sec x \tan x}}$$

Derivatives of Trigonometric Functions:

$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\csc x] = -\csc x \cot x$
$\frac{d}{dx}[\cos x] = -\sin x$	$\frac{d}{dx}[\sec x] = \sec x \tan x$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\frac{d}{dx}[\cot x] = -\csc^2 x$

Now, let's practice! We'll use all the derivative techniques we've studied so far: Power, Product, Quotient, and Chain Rule along with trigonometric functions. Here we go...

Find the derivative of each function. Simplify, if possible.

EX #5: $y = 4 \sin x$

$$\underline{\underline{y' = 4 \cos x}}$$

EX #6: $y = \sin 4x$

$$y' = [\cos(4x)](4)$$

$$\underline{\underline{y' = 4 \cos 4x}}$$

EX #7: $y = \sin^2 x$

$$y = (\sin x)^2$$

$$y' = 2(\sin x)(\cos x)$$

$$\underline{\underline{y' = 2 \sin x \cos x}}$$

EX #8: $y = \sin^2(4x)$

$$y = (\sin 4x)^2$$

$$y' = 2(\sin 4x)(\cos 4x)(4)$$

$$\underline{\underline{y' = 8(\sin 4x)(\cos 4x)}}$$

EX #9: $y = \sin \sqrt{x}$

$$y = \sin(x^{1/2})$$

$$y' = (\cos \sqrt{x})(\frac{1}{2}x^{-1/2})$$

$$\underline{\underline{y' = \frac{\cos \sqrt{x}}{2\sqrt{x}}}}$$

EX #10: $y = \sqrt{\sin x}$

$$y = (\sin x)^{1/2}$$

$$y' = \frac{1}{2}(\sin x)^{-1/2}(\cos x)$$

$$\underline{\underline{y' = \frac{\cos x}{2\sqrt{\sin x}}}}$$

EX #11: $y = \sqrt{\sin(4x)}$

$$y = (\sin 4x)^{1/2}$$

$$y' = \frac{1}{2}(\sin 4x)^{-1/2}(\cos 4x)(4)$$

$$\underline{\underline{y' = \frac{2 \cos 4x}{\sqrt{\sin 4x}}}}$$

EX #12: $y = \sin x \cos x$

$$y' = \sin x(-\sin x) + \cos x(\cos x)$$

$$\underline{\underline{y' = \cos^2 x - \sin^2 x}}$$

EX #13: $y = x \cos x$

$$y' = x(-\sin x) + (\cos x)(1)$$

$$\underline{\underline{y' = \cos x - x \sin x}}$$

EX #14: $y = \sin x + \cos x$

$$\underline{\underline{y' = \cos x - \sin x}}$$

EX #15: $y = 4 \cos x^2$

$$y' = 4(-\sin x^2)(2x)$$

$$\underline{\underline{y' = -8x \sin x^2}}$$

EX #16: $y = 4 \cos^2 x$

$$y = 4(\cos x)^2$$

$$y' = 8(\cos x)(-\sin x)$$

$$\underline{\underline{y' = -8 \sin x \cos x}}$$

EX #17: $y = \sqrt{\cos 2x}$

$$y = (\cos 2x)^{1/2}$$

$$y' = \frac{1}{2}(\cos 2x)^{-1/2}(-\sin 2x)(2)$$

$$\underline{\underline{y' = \frac{-\sin 2x}{\sqrt{\cos 2x}}}}$$

EX #18: $y = \tan \sqrt{x}$

$$y = \tan x^{1/2}$$

$$y' = (\sec^2 \sqrt{x})\left(\frac{1}{2}x^{-1/2}\right)$$

$$\underline{\underline{y' = \frac{\sec^2 \sqrt{x}}{2\sqrt{x}}}}$$

EX #19: $y = \tan^2 x$

$$y = (\tan x)^2$$

$$y' = 2(\tan x)(\sec^2 x)$$

$$\underline{\underline{y' = 2 \sec^2 x \tan x}}$$

EX #20: $y = \sec^2(x-1)$

$$y = [\sec(x-1)]^2$$

$$y' = 2 \sec(x-1)[\sec(x-1)\tan(x-1)]$$

$$\underline{\underline{y' = 2 \sec^2(x-1)\tan(x-1)}}$$

EX #21: $y = -4 \csc(2x)$

$$y' = -4(-\csc 2x \cot 2x)(2)$$

$$\underline{\underline{y' = 8 \csc(2x) \cot(2x)}}$$

EX #22: $y = \sqrt[3]{\cos x}$

$$y = (\cos x)^{1/3}$$

$$y' = \frac{1}{3}(\cos x)^{-2/3}(-\sin x)$$

$$\underline{\underline{y' = \frac{-\sin x}{3(\cos x)^{2/3}}}}$$

EX #23: $y = \frac{x^3}{\cos x}$

$$y' = \frac{(\cos x)(3x^2) - x^3(-\sin x)}{\cos^2 x}$$

$$\underline{\underline{y' = \frac{3x^2 \cos x + x^3 \sin x}{\cos^2 x}}}$$

EX #24: $y = \sqrt{\tan 3x}$

$$y = (\tan 3x)^{1/2}$$

$$y' = \frac{1}{2}(\tan 3x)^{-1/2}(\sec^2 3x)(3)$$

$$\underline{\underline{y' = \frac{3 \sec^2 3x}{2 \sqrt{\tan 3x}}}}$$

$$15) \quad y = \frac{\sin x}{1 + \cos x}$$

$$y' = \frac{(1 + \cos x)(\cos x) - (\sin x)(-\sin x)}{(1 + \cos x)^2}$$

$$y' = \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$y' = \frac{\cancel{\cos x} + \cancel{1}}{(1 + \cos x)^2} = \frac{1}{1 + \cos x}$$

$$12) \quad y = \sec(\pi x)$$

$$y' = \sec(\pi x) \tan(\pi x) \cdot \pi$$

$$y' = \pi \sec(\pi x) \tan(\pi x)$$

$$21) g(x) = \sin^2 x = (\sin x)^2$$

$$g'(x) = 2 \boxed{(\sin x)} \cdot \boxed{(\cos x)}$$

$$g''(x) = 2 \left((\sin x)(-\sin x) + (\cos x)(\cos x) \right)$$
$$= \boxed{2(-\sin^2 x + \cos^2 x)}$$

$$= -2\sin^2 x + 2\cos^2 x$$

$$y = 2x^4$$

$$y' = 2 \cdot 4x^3$$

$$= 8x^3$$

$$22) f(x) = \sin(2x) + 2\cos x$$

$$f'(x) = 2\cos(2x) + 2(-\sin x)$$

$$0 = 2(\cos(2x) - \sin x)$$

$$0 = \cos(2x) - \sin x$$

$$\sin x = \cos(2x)$$

$$x + 2x = 90$$

$$3x = 90$$

$$x = 30$$